SUPPLEMENTAL PROBLEMS QUESTIONS

LEVEL 1: NUMBER THEORY

1. Which of the following numbers is greater than 0.216?

   (A) 0.2106
   (B) 0.215
   (C) 0.2156
   (D) 0.2159
   (E) 0.2161

2. What is the greatest positive integer that is a divisor of 14, 49, and 63?

   (A) 14
   (B) 7
   (C) 3
   (D) 2
   (E) 1

3. A positive integer is called a palindrome if it reads the same forward as it does backward. For example, 2442 is a palindrome. What is the smallest palindrome greater than 2513?

   (A) 2112
   (B) 2514
   (C) 2525
   (D) 2552
   (E) 2553

4. Which of the following numbers is between $\frac{1}{8}$ and $\frac{1}{7}$?

   (A) 0.13
   (B) 0.15
   (C) 0.17
   (D) 0.19
   (E) 0.21

5. Which of the following numbers disproves the statement "A number that is divisible by 6 and 12 is also divisible by 18"?

   (A) 18
   (B) 36
   (C) 48
   (D) 72
   (E) 108

6. $(4+5)^2 =$

7. If 4.56 is rounded to the nearest tenth and the result is doubled, what is the final result?

8. Three consecutive integers are listed in increasing order. If their sum is 603, what is the third integer in the list?

LEVEL 1: ALGEBRA AND FUNCTIONS

9. If $1+x+2+x+3=x+1+x+2+x$, what is the value of x?

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

10. If $a+b=12$ and $a=5$, then $4b=$

(A) 12
(B) 21
(C) 24
(D) 28
(E) 35

11. If Robert drove a miles in b hours, which of the following represents his average speed, in miles per hour?

(A) $\frac{a}{b}$
(B) $\frac{b}{a}$
(C) $\frac{1}{ab}$
(D) ab
(E) a^2b

12. If $3y-18=15$, then $y-6=$

(A) 30
(B) 20
(C) 15
(D) 10
(E) 5

13. If $4^2=2^z$, then $z=$

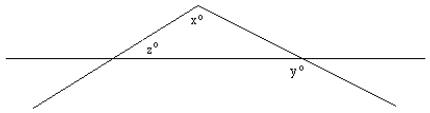
(A) 4
(B) 3
(C) 2
(D) 1
(E) 0

14. If $a=\frac{7}{b}$ and $c=8a$, what is the value of c when $b=42$?

15. If $(c+2)(15-6)=21$, then $5c=$

16. If $7(x-5)=6(x-4)$, what is the value of x?

LEVEL 1: GEOMETRY



Note: Figure not drawn to scale.

17. In the figure above, if $y=130$ and $z=55$, what is the value of x?

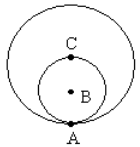
(A) 15
(B) 30
(C) 50
(D) 75
(E) 130

18. If the degree measures of the three angles of a triangle are 60°, z° and z°, what is the value of z?

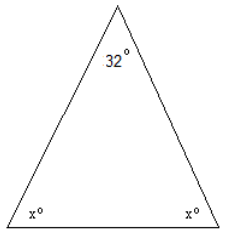
(A) 60
(B) 65
(C) 70
(D) 75
(E) 80

19. The volume of a rectangular box is 6 cubic inches. If the width of the box is 14 inches and the height is $\frac{1}{7}$ inch, what is the length?

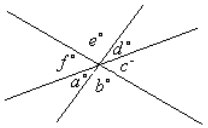
(A) $\frac{1}{14}$ inch
(B) $\frac{1}{7}$ inch
(C) 1 inch
(D) 3 inches
(E) 4 inches



20. In the figure above, A, B, and C lie on the same line. B Is the center of the smaller circle, and C is the center of the larger circle. If the radius of the smaller circle is 7, what is the diameter of the larger circle?

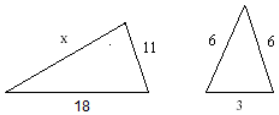


21. In the triangle above, what is the value of x?



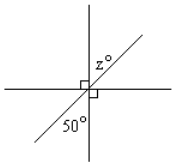
Note: Figure not drawn to scale.

22. In the figure above, three lines intersect at a point. If $a = 27$ and $c = 61$, what is the value of e?



Note: Figure not drawn to scale.

23. If the perimeter of the triangle on the left is three times the perimeter of the triangle on the right, what is the value of x?



24. In the figure above, what is the value of z?

LEVEL 1: PROBABILITY AND STATISTICS

25. The average (arithmetic mean) of four numbers is 100. If three of the numbers are 60, 70 and 140, what is the fourth number?

    (A) 90
    (B) 100
    (C) 110
    (D) 120
    (E) 130

26. A menu lists 8 meals and 3 drinks. How many different meal-drink combinations are possible from this menu?

    (A) 24
    (B) 12
    (C) 11
    (D) 8
    (E) 3

3, 4, 5, 6, 7, 8, 9, 10, 11

27. If a number is selected at random from the list above, what is the probability that it will be less than 7?

    (A) $\frac{2}{9}$
    (B) $\frac{1}{3}$
    (C) $\frac{4}{9}$
    (D) $\frac{2}{3}$
    (E) 1

28. There are exactly 17 coins in a bag. There are 6 pennies, 4 nickels, 5 dimes, and the rest are quarters. If one coin is selected at random from the bag, what is the probability that the coin is a quarter?

29. Fred's math class has a homework list of 7 algebra problems and 4 geometry problems. Fred will select one algebra problem and one geometry problem from the list to complete the homework assignment. How many different choices of an algebra problem and a geometry problem are possible?

30. The average (arithmetic mean) of eight numbers is 250. If the sum of seven of the numbers is 1463, what is the eighth number?

31. In a jar there are exactly 56 marbles, each of which is yellow, purple, or blue. The probability of randomly selecting a yellow marble from the jar is $\frac{2}{7}$, and the probability of randomly selecting a purple marble from the jar is $\frac{3}{7}$. How many marbles in the jar are blue?

32. Joe, Mike, Phil, John and Fred own a total of 138 CDs. If John owns 38 of them, what is the average (arithmetic mean) number of CDs owned by Joe, Mike, Phil and Fred?

LEVEL 2: NUMBER THEORY

33. Starting with a blue tile, colored tiles are placed in a row according to the pattern blue, green, yellow, orange, red, purple. If this pattern is repeated, what is the color of the 73rd tile?

    (A) Blue
    (B) Green
    (C) Yellow
    (D) Orange
    (E) Red

34. If k is an even integer, what is the smallest even integer greater than k ?

(A) $k+3$
(B) $k+2$
(C) $k+1$
(D) $2(k+1)$
(E) $2(k+1)+3$

35. A room has 2000 square feet of surface that needs to be painted. If 3 gallons of paint will cover 520 square feet, what is the least whole number of gallons that must be purchased in order to have enough paint to cover the entire surface?

(A) 8
(B) 9
(C) 10
(D) 11
(E) 12

36. A piece of cable x feet in length is cut into exactly 6 pieces, each 2 feet 3 inches in length. What is the value of x ?

(A) $12\frac{2}{3}$
(B) 13
(C) $13\frac{1}{2}$
(D) $13\frac{2}{3}$
(E) 14

37. In September, Maria was able to type 30 words per minute. In October she was able to type 42 words per minute. By what percent did Jennifer's speed increase from September to October?

(A) 12%
(B) 18%
(C) 30%
(D) 40%
(E) 42%

20, 59, 30, 89, 45, ...

38. In the sequence above, 20 is the first term and each term thereafter is obtained by using the following rules.

- If the previous term is even, multiply it by 3 and then subtract 1.
- If the previous term is odd, add 1 to it and then divide by 2.

What is the eighth term of the sequence?

39. A copy machine makes 2000 copies per hour. At this rate, in how many minutes can the copy machine produce 800 copies?

40. The ratio of 17 to 3 is equal to the ratio of 102 to what number?

LEVEL 2: ALGEBRA AND FUNCTIONS

41. If $3x+7=24$, then $3x-7=$

(A) 8
(B) 9
(C) 10
(D) 11
(E) 12

42. The number a is 5 less than 7 times the number b. The sum of a and b is 14. Which of the following pairs of equations could be used to find the values of a and b?

(A) $a = 7(b-5)$
$a + b = 14$

(B) $a = 5(7-b)$
$a = 14 - b$

(C) $a = 7(b-5)$
$a = 14 - b$

(D) $a = 7b - 5$
$a + b = 14$

(E) $a = 5 - 7b$
$a + b = 14$

43. Which of the following is an expression for 25 less than the product of y and 7?

(A) $7y - 25$
(B) $25 - 7y$
(C) $(y+7) - 25$
(D) $7(y-25)$
(E) $y^7 - 25$

44. If $\frac{y}{z} = -3$, then $y + 3z =$

(A) -1
(B) 0
(C) 1
(D) y
(E) z

45. If $7x + y = 6$ and $5x + y = 2$, what is the value of $6x + y$?

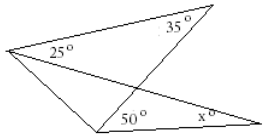
(A) -8
(B) 4
(C) 6
(D) 12
(E) 18

46. If 3 less than x is 1 more than 6, what is the value of x?

47. If $6^{x+1} = 7776$, what is the value of x?

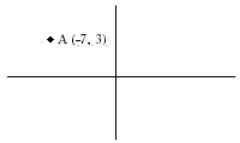
48. If $6x - 2y = 11$, what is the value of $4(6x - 2y)$?

LEVEL 2: GEOMETRY



49. In the figure above, what is the value of x?

(A) 10
(B) 15
(C) 20
(D) 25
(E) 30



50. In the figure above, a line is to be drawn through point A so that it never crosses the y-axis. Through which of the following points must the line pass?

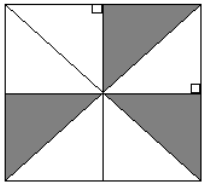
(A) $(7, 3)$
(B) $(7, -3)$
(C) $(-7, -3)$
(D) $(3, 7)$
(E) $(-3, -7)$

51. C is the midpoint of line segment AB, and D and E are the midpoints of AC and CB, respectively. If the length of DE is 7, what is the length of AB?

(A) 3.5
(B) 7
(C) 10.5
(D) 14
(E) 17.5

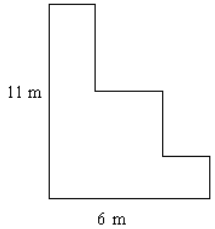
52. In an xy coordinate system, which point lies in the interior of a circle with center (0, 0) and radius 4?

(A) (1, -4)
(B) (-2, -3)
(C) (-4, 1)
(D) (0, 4)
(E) (4, 4)



53. What percent of the square above is shaded?

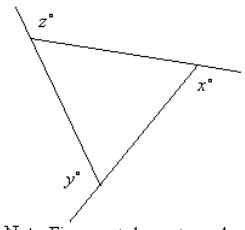
(A) 25%
(B) $33\frac{1}{3}$%
(C) $37\frac{1}{2}$%
(D) 50%
(E) $62\frac{2}{3}$%



Note: Figure not drawn to scale.

54. What is the perimeter, in meters, of the figure above?

55. If the sum of the areas of two congruent squares is 72, what is the length of a side of each square?



Note: Figure not drawn to scale.

56. In the figure above, what is the value of $x + y + z$?

LEVEL 2: PROBABILITY AND STATISTICS

57. The average (arithmetic mean) of three numbers is 72. If one of the numbers is 21, what is the sum of the other two?

(A) 71
(B) 92
(C) 177
(D) 183
(E) 195

58. What is the average (arithmetic mean) of $11-k$, 11, and $11+k$?

(A) 3
(B) 11
(C) 15
(D) $3+\frac{k}{3}$
(E) $11+\frac{k}{3}$

59. The average (arithmetic mean) of 9, 15, 18, and y is 20. What is the value of y ?

(A) 25
(B) 36
(C) 38
(D) 39
(E) 67

16, 21, 13, 12, 3, 19, 2, 5, 22, 36, b

60. If b is the median of the 11 numbers listed above, which of the following could be the value of b ?

(A) 4
(B) 8
(C) 14
(D) 17
(E) 18

61. The average (arithmetic mean) of ten numbers is 77. If an eleventh number, 22, is added to the group, what is the average of the eleven numbers?

62. A is a set of numbers whose average (arithmetic mean) is 11. B is a set that is generated by doubling each number in A. What is the average of the numbers in set B ?

63. The average (arithmetic mean) of 7, 11, 15, and z is z . What is the value of z ?

64. Of the marbles in a jar, 14 are green. Joseph randomly takes one marble out of the jar. If the probability is $\frac{7}{8}$ that the marble he chooses is green, how many marbles are in the jar?

LEVEL 3: NUMBER THEORY

65. If an integer n is divisible by 5, 7, 25, and 35, what is the next larger integer divisible by these numbers?

(A) $n+50$
(B) $n+75$
(C) $n+125$
(D) $n+150$
(E) $n+175$

66. If x and y are negative odd integers, which of the following must be a negative odd integer?

(A) $x+y$
(B) $x-y$
(C) $2x+y$
(D) $2x-y$
(E) $\frac{x+y}{2}$

67. How many seconds are required for a bicycle to go 3 miles at a constant speed of 6 miles per hour?

(A) 3600
(B) 3000
(C) 2400
(D) 1800
(E) 1200

68. The cost of 7 hats is d dollars. At this rate, what is the cost, in dollars of 63 hats?

(A) $\frac{9d}{7}$
(B) $\frac{d}{63}$
(C) $\frac{63}{d}$
(D) $9d$
(E) $63d$

69. When 407 is divided by 9, the remainder is r, and when 234 is divided by 7, the remainder is s. What is the value of rs?

70. What is the smallest positive integer divisible by 3, 5, 7, and 35?

71. What percent of 70 is 14? (Disregard the percent symbol when gridding in your answer.)

72. The quantity (3×4^{11}) is how many times the quantity (3×4^{6})?

LEVEL 3: ALGEBRA AND FUNCTIONS

73. In the exact middle of a certain book, when the page numbers on the facing pages, x and $x+1$, are multiplied together, the product is 272. If all the pages are numbered in order, how many numbered pages are in the book?

(A) 30
(B) 32
(C) 34
(D) 36
(E) 38

74. If x is $\frac{3}{5}$ of y and y is $\frac{5}{7}$ of z, what is the value of $\frac{z}{x}$?

(A) $\frac{1}{4}$
(B) $\frac{3}{7}$
(C) $\frac{5}{4}$
(D) $\frac{10}{7}$
(E) $\frac{7}{3}$

75. If $\frac{9}{x}+\frac{7}{8}=\frac{85}{56}$, what is the value of x?

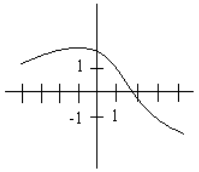
(A) -14
(B) -7
(C) 2
(D) 7
(E) 14

76. If $20x+36y=56$, what is the value of $5x+9y$?

(A) 7
(B) 8
(C) 14
(D) 15
(E) 30

77. The function k is defined by $k(x)=3x^2+bx-2$, where b is a constant. In the xy-plane, the graph of $y=k(x)$ crosses the x-axis where $x=2$. What is the value of b?

(A) 5
(B) 2.5
(C) 0
(D) -2.5
(E) -5



78. The figure above shows the graph of the function f. Which of the following is less than $f(2)$?

(A) $f(-4)$
(B) $f(-3)$
(C) $f(-1)$
(D) $f(1)$
(E) $f(3)$

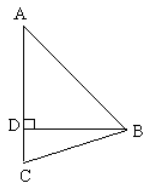
79. If $7^x=5$, then $7^{4x}=$

80. Let g be a function such that $g(x)=|3x|-k$ where k is a constant. If $g(4)=-2$, what is the value of $g(-5)$?

LEVEL 3: GEOMETRY

81. The volume of a right circular cylinder is 216π cubic centimeters. If the height and base radius of the cylinder are equal, what is the base diameter of the cylinder?

(A) 4 centimeters
(B) 6 centimeters
(C) 10 centimeters
(D) 12 centimeters
(E) 36 centimeters



Note: Figure not drawn to scale.

82. Triangle ABC has the same area as a rectangle with sides of lengths 3 and 11. If the length of BD is 6, what is the length of AC?

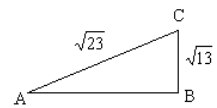
(A) 8
(B) 9
(C) 10
(D) 11
(E) 12

83. In the xy-coordinate plane, line n passes through the points (0,3) and (-1,0). If line m is perpendicular to line n, what is the slope of line m?

(A) - 3
(B) $-\frac{1}{3}$
(C) 1
(D) $\frac{1}{3}$
(E) 3

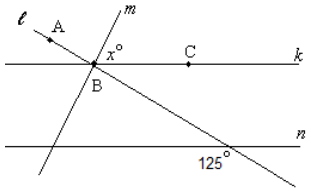
84. Point O lies in plane P. How many circles are there in plane P that have center O and an area of 25π centimeters?

(A) None
(B) One
(C) Two
(D) Three
(E) More than three



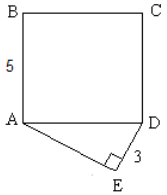
Note: Figure not drawn to scale.

85. In right triangle ABC above, what is the length of side AB to the nearest tenth?

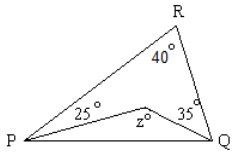


Note: Figure not drawn to scale.

86. In the figure above, line k is parallel to line n. If line m bisects angle ABC, what is the value of x?



87. In the figure above, $ABCD$ is a square. What is the area of triangle AED?



Note: Figure not drawn to scale.

88. In triangle PQR above, what is the value of z?

LEVEL 3: PROBABILITY AND STATISTICS

89. A chemist is testing 6 different liquids. For each test, the chemist chooses 3 of the liquids and mixes them together. What is the least number of tests that must be done so that every possible combination of liquids is tested?

(A) 5
(B) 10
(C) 15
(D) 20
(E) 25

90. There are y bricks in a row. If one brick is to be selected at random, the probability that it will <u>not</u> be cracked is $\frac{6}{7}$. In terms of y, how many of the bricks are cracked?

(A) $\frac{y}{7}$
(B) $\frac{5y}{7}$
(C) $\frac{7y}{5}$
(D) $\frac{12y}{7}$
(E) $7y$

TEST GRADES OF STUDENTS IN MATH CLASS

Test Grade	65	81	84	92	100
Number of students with that grade	1	8	6	2	4

91. The test grades of the 21 students in a math class are shown in the chart above. What is the median test grade for the class?

    (A) 65
    (B) 81
    (C) 82.5
    (D) 84
    (E) 92

92. A group of dogs are fed Brand A and Brand B dog food only. Of these dogs, 4 dogs eat Brand A and 9 dogs eat Brand B. If 2 of the dogs that eat Brand B also eat Brand A, how many dogs are in the group?

93. The average (arithmetic mean) of eleven numbers is 15. When a twelfth number is added, the average of the twelve numbers is 14. What is the twelfth number?

94. Set A contains only the integers 0 through 157 inclusive. If a number is selected at random from A, what is the probability that the number selected will be less than 79?

95. Seven different books are to be stacked in a pile. One book is chosen for the bottom of the pile. In how many different orders can the remaining books be placed on the stack?

96. The average of x, y, z, and w is 8 and the average of z and w is 5. What is the average of x and y?

LEVEL 4: NUMBER THEORY

97. If the ratio of two positive integers is 5 to 4, which of the following statements about these integers CANNOT be true?

    (A) Their sum is an odd integer.
    (B) Their sum is an even integer.
    (C) Their product is divisible by 7.
    (D) Their product is an even integer.
    (E) Their product is an odd integer.

98. A positive integer is called a palindrome if it reads the same forward as it does backward. For example, 2442 is a palindrome. How many four-digit palindromes are there?

    (A) 19
    (B) 20
    (C) 90
    (D) 100
    (E) 810

99. The sum of 8 positive even integers is 158. If each of these integers is distinct, what is the greatest possible value of one of these integers?

    (A) 106
    (B) 104
    (C) 102
    (D) 100
    (E) 98

100. A person cuts a cake into n equal pieces and eats five pieces. In terms of n, what percent of the cake has been eaten?

(A) 500%
(B) $\frac{500}{n}\%$
(C) $\frac{100n}{5}\%$
(D) $\frac{n-5}{100}\%$
(E) $\frac{100(n-5)}{n}\%$

101. If m, and n are distinct positive integers such that n is divisible by m, and m is divisible by 3, which of the following statements must be true?

I. n is divisible by 3.
II. $n = 3m$.
III. n has more than 3 positive factors.

(A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III

102. Set A consists of k integers, and the difference between the greatest integer in A and the least integer in A is 500. A new set of k integers, set B, is formed by multiplying each integer in A by 6 and then subtracting 10 from the product. What is the difference between the greatest integer in B and the least integer in B?

(A) 500
(B) 2490
(C) 2500
(D) 2990
(E) 3000

103. If k is divided by 8, the remainder is 3. What is the remainder if $3k$ is divided by 8?

104. A mixture is made by combining a red liquid and a blue liquid so that the ratio of the red liquid to the blue liquid is 19 to 4 by weight. How many liters of the blue liquid are needed to make a 552 liter mixture?

LEVEL 4: ALGEBRA AND FUNCTIONS

105. If $x^2 = 4$ and $y^2 = 7$, then $(2x + y)^2$ could equal which of the following?

(A) 23
(B) 65
(C) 121
(D) $23 - 8\sqrt{7}$
(E) $65 + 8\sqrt{7}$

106. If $y = 3^x$, which of the following expressions is equivalent to $9^x - 3^{x+2}$ for all positive integer values of x?

(A) $3y - 3$
(B) y^2
(C) $y^2 - y$
(D) $y^2 - 3y$
(E) $y^2 - 9y$

107. Positive integers a, b, and c satisfy the equations $a^{-b} = \frac{1}{64}$ and $b^c = 216$. If $a < b$, what is the value of $2a + b - c$?

(A) 6
(B) 7
(C) 8
(D) 9
(E) 10

108. A small hotel has 15 rooms which are all occupied. If each room is occupied by either one or two guests and there are 27 guests in total, how many rooms are occupied by two guests?

109. Let $\bullet$ be defined by $x \bullet y = y^x$. If $a = x \bullet 2$, $b = y \bullet 2$, and $x + y = 3$, what is the value of ab?

110. Let the function f be defined for all values of x by $f(x) = x(x+1)$. If k is a positive number and $f(k+5) = 72$, what is the value of k?

111. For any real numbers r and s such that $r \neq s$, let $r \propto s$ be defined by $r \propto s = \frac{r+s}{r-s}$. If $r + s = 49$ and $r \propto s = 7$, what is the value of s?

112. For all numbers a and b, let $a \Diamond b = a^2 - 2ab^3$. What is the value of $|(4 \Diamond 1) \Diamond 2|$?

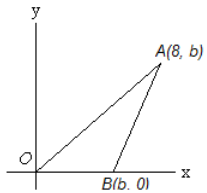
LEVEL 4: GEOMETRY

113. Point A is a vertex of a 7-sided polygon. The polygon has 7 sides of equal length and 7 angles of equal measure. When all possible diagonals are drawn from point A in the polygon, how many triangles are formed?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

114. If $a > 1$, what is the slope of the line in the xy-plane that passes through the points (a, a^3) and (a^2, a^5)?

(A) $-a^3 + 6a^2$
(B) $-a^3 + a^2$
(C) $-a^3 - a^2$
(D) $a^3 - a^2$
(E) $a^3 + a^2$



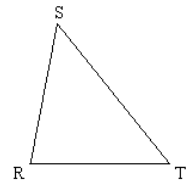
Note: Figure not drawn to scale.

115. In the xy-plane above, the area of triangle OAB is 32. What is the value of b?

(A) 8
(B) 12
(C) 16
(D) 32
(E) 64

116. A container in the shape of a right circular cylinder has an inside base diameter of 12 centimeters and an inside height of 10 centimeters. This cylinder is completely filled with fluid. All of the fluid is then poured into a second right circular cylinder with a smaller inside base diameter of 8 centimeters. What must be the minimum inside height, in centimeters, of the second container?

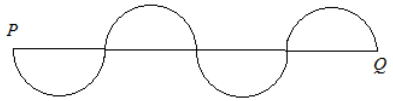
117. Points P and Q are on the surface of a sphere that has a volume of 972π cubic meters. What is the greatest possible length, in meters, of line segment PQ? (The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)



Note: Figure not drawn to scale.

118. In the triangle above, $RS = RT = 15$ and $ST = 8$. What is the area of the triangle to the nearest tenth?

119. In the xy-plane, line ℓ is the graph of $5x + ky = 9$, where k is a constant. The graph of $20x + 16y = 11$ is parallel to line ℓ. What is the value of k?



120. In the figure above, the diameters of the four semicircles are equal and lie on line segment PQ. If the length of line segment PQ is $\frac{64}{\pi}$, what is the length of the curve from P to Q?

LEVEL 4: PROBABILITY AND STATISTICS

121. If the average (arithmetic mean) of a, b, and 26 is 16, what is the average of a and b?

(A) $5\frac{1}{2}$
(B) 8
(C) 11
(D) 22
(E) It cannot be determined from the information given.

122. Twenty one people were playing a game. 6 people scored 10 points, 5 people scored 30 points, 4 people scored 50 points, 3 people scored 70 points, 2 people scored 90 points, and 1 person scored 100 points. Which of the following correctly shows the order of the median, mode and average (arithmetic mean) of the 21 scores?

(A) average < median< mode
(B) average < mode < median
(C) median < mode < average
(D) median < average < mode
(E) mode < median < average

123. The average (arithmetic mean) of 5 numbers is x. If one of the numbers is y, what is the average of the remaining 4 numbers in terms of x and y?

(A) $\frac{x}{5}$
(B) $5x-y$
(C) $\frac{4x-y}{5}$
(D) $\frac{5x-y}{4}$
(E) $\frac{5y-x}{4}$

124. An urn contains a number of marbles of which 63 are green, 15 are purple, and the remainder are orange. If the probability of picking an orange marble from this urn at random is $\frac{1}{3}$, how many orange marbles are in the urn?

(A) 13
(B) 26
(C) 39
(D) 78
(E) 244

125. Exactly 5 actors try out for the 5 parts in a movie. If each actor can perform any one part and no one will perform more than one part, how many different assignments of actors are possible?

126. A wall is to be painted one color with a stripe of a different color running through the middle. If 8 different colors are available, how many color combinations are possible?

127. A pet store has a white dog, a black dog, and a grey dog. The store also has three cats – one white, one black, and one grey – and three birds – one white, one black, and one grey. Jonathon wants to buy one dog, one cat, and one bird. How many different possibilities does he have?

128. Any 2 points determine a line. If there are 10 points in a plane, no 3 of which lie on the same line, how many lines are determined by pairs of these 10 points?

LEVEL 5: NUMBER THEORY

129. In an empty square field, k rows of k trees are planted so that the whole field is filled with trees. If n of these trees lie along the boundary of the field, which of the following is equal to n?

(A) $k^2-(k-3)^2$
(B) $k^2-(k-2)^2$
(C) $k^2-(k-1)^2$
(D) $4k-3$
(E) $4k-2$

130. If x and y are integers and x^2y+xy^2 is even, which of the following statements must be true?

I. x is even
II. xy is even
III. $x+y$ is even

(A) None
(B) I only
(C) III only
(D) I and III only
(E) I, II, and III

131. If $a^{14}b^{15}c^{16}d^{17}>0$, which of the following products must be positive?

(A) ab
(B) ac
(C) bc
(D) ad
(E) bd

132. If $a_k=5+5^2+5^3+5^4+\cdots+5^k$, for which of the following values of k will a_k be divisible by 10?

(A) 37
(B) 51
(C) 75
(D) 88
(E) 91

133. If n is a positive integer and $k=(n^3-n)^2$, which of the following statements about k must be true for all values of n?

I. k is a multiple of 3
II. k is a multiple of 4
III. k is a multiple of 36

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

134. How many positive integers less than 3,000 are multiples of 13 and are equal to 3 times an even integer?

135. In how many of the integers from 1 to 150 does the digit 4 appear at least once?

136. In Steve's math class, 12 students play the piano and 17 students play the guitar. If a total of 19 students play only one of these two instruments, how many students play both instruments?

LEVEL 5: ALGEBRA AND FUNCTIONS

137. For how many integers n is $(7n-26)(5n+11)$ a negative number?

(A) None
(B) Two
(C) Four
(D) Six
(E) Eight

$$x = 25z$$
$$y = 25z^2 + 3$$

138. If $z > 0$ in the equation above, what is y in terms of x?

(A) $\frac{1}{25}x^2 + 1$
(B) $\frac{1}{25}x^2 + 2$
(C) $\frac{1}{25}x^2 + 3$
(D) $\frac{1}{5}x^2 + 2$
(E) $x^2 + 3$

139. If a and b are positive integers, which of the following is equivalent to $(5a)^{3b} - (5a)^{2b}$?

(A) $(5a)^b$
(B) $5^b(a^3 - a^2)$
(C) $(5a)^{2b}[(5a)^{3b} - 1]$
(D) $(5a)^{2b}(25a - 1)$
(E) $(5a)^{2b}[(5a)^b - 1]$

$$k = a - b + 16$$
$$k = b - c - 7$$
$$k = c - d - 9$$
$$k = d - e + 12$$
$$k = e - a - 2$$

140. In the system of equations above, what is the value of k?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

141. If $x^2 - y^2 = 10 - k - 3k^2$, and $x - y = 5 - 3k$, what is $x + y$ in terms of k?

(A) $k - 2$
(B) $(k-2)^2$
(C) $k + 2$
(D) $(k+2)^2$
(E) $k^2 - 4$

x	5	10	15
$g(x)$	4	a	10

x	7	14	28
$h(x)$	2	b	14

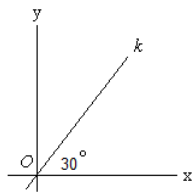
142. The tables above show some values for the functions g and h. If g and h are linear functions, what is the value of $5a - 2b$?

143. Let g be a function such that $g(x)=k(x-7)(x+7)$ where k is a nonzero constant. If $g(b-5.3)=0$ and $b>0$, what is the value of b?

$$(x-6)(x-2n)=x^2-8nx+k$$

144. In the equation above, n and k are constants. If the equation is true for all values of x, what is the value of k?

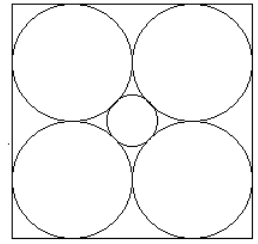
LEVEL 5: GEOMETRY



Note: Figure not drawn to scale.

145. In the figure above, what is the equation of line k?

(A) $y=\frac{x}{2}$
(B) $y=\frac{x}{\sqrt{2}}$
(C) $y=\frac{x}{\sqrt{3}}$
(D) $y=\sqrt{2}x$
(E) $y=\sqrt{3}x$

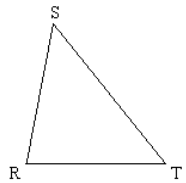


146. In the figure above, each of the four large circles is tangent to two of the other large circles, the small circle, and two sides of the square. If the diameter of each of the large circles is 10, what is the radius of the small circle?

(A) $5\sqrt{2}$
(B) 5
(C) $5\sqrt{2}-5$
(D) $\frac{5}{2}$
(E) $5\sqrt{2}-1$

147. The lengths of the sides of an isosceles triangle are 22, n, and n. If n is an integer, what is the smallest possible perimeter of the triangle?

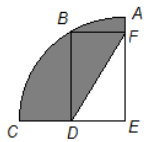
(A) 30
(B) 31
(C) 32
(D) 34
(E) 46



Note: Figure not drawn to scale.

148. In the triangle above, $RS = 4$ and $ST = 6$. Point U lies on RT between R and T so that $SU \perp RT$. Which of the following can be the length of SU?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 8



Note: Figure not drawn to scale.

149. In the figure above, arc ABC is one quarter of a circle with center E and radius $8\sqrt{2}$. If the length plus the width of rectangle $BDEF$ is 16, then the area of the shaded region is

(A) $32\pi - 32$
(B) $32\pi - 16$
(C) $32\pi - 8$
(D) $32\pi + 16$
(E) $32\pi + 32$

150. If the length of a rectangle is increased by 60%, and the width of the same rectangle is decreased by 50%, then the area of the rectangle is decreased by $x\%$. What is the value of x?

151. In the $xy-$plane, line k has equation $y = \frac{12}{7}x + b$, and line n has equation $y = \frac{3}{2}x + 4$. If the lines intersect at the point with coordinates $(a, \frac{9}{4})$, what is the value of b?

152. The lengths of the sides of a triangle are x, 12, and 23, where x is the shortest side. If the triangle is not isosceles, what is a possible value of x?

LEVEL 5: PROBABILITY AND STATISTICS

$$\frac{1}{x^3}, \frac{1}{x^2}, \frac{1}{x}, x^2, x^3$$

153. If $-1 < x < 0$, what is the median of the five numbers in the list above?

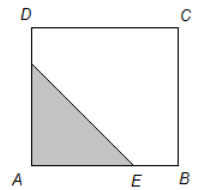
(A) $\frac{1}{x^3}$
(B) $\frac{1}{x^2}$
(C) $\frac{1}{x}$
(D) x^2
(E) x^3

154. The integers 1 through 7 are written on each of seven cards. The cards are shuffled and one card is drawn at random. That card is then replaced, the cards are shuffled again and another card is drawn at random. This procedure is repeated one more time (for a total of three times). What is the probability that the sum of the numbers on the three cards drawn was 20 or 21?

(A) $\frac{1}{343}$
(B) $\frac{2}{343}$
(C) $\frac{4}{343}$
(D) $\frac{1}{49}$
(E) $\frac{2}{49}$

155. If $g = a+b+c+d+e+f$, what is the average (arithmetic mean) of a, b, c, d, e, f and g in terms of g?

(A) $\frac{3g}{7}$
(B) $\frac{2g}{5}$
(C) $\frac{g}{3}$
(D) $\frac{2g}{7}$
(E) $\frac{g}{4}$



Note: Figure not drawn to scale.

156. In the figure above, $ABCD$ is a square, $AD = 8$, $EB = 8-4b$, and the triangle is isosceles. A point in square $ABCD$ is to be chosen at random. If the probability that the point will be in the shaded triangle is $\frac{1}{4}$, what is the value of b?

(A) $\sqrt{2}$
(B) $\sqrt{3}$
(C) $2\sqrt{2}$
(D) $2\sqrt{3}$
(E) 4

157. A five digit number is to be formed using each of the digits 1, 2, 3, 4, and 5 exactly once. How many such numbers are there in which the digits 1 and 2 are next to each other?

(A) 36
(B) 48
(C) 60
(D) 72
(E) 120

158. Let a, b and c be numbers with $a<b<c$ such that the average of a and b is 4, the average of b and c is 8, and the average of a and c is 6. What is the average of a, b and c?

159. The average (arithmetic mean) salary of employees at a bank with A employees in thousands of dollars is 55, and the average salary of employees at a bank with B employees in thousands of dollars is 95. When the salaries of both banks are combined, the average salary in thousands of dollars is 71. What is the value of $\frac{A}{B}$?

160. How many integers between 5000 and 8000 have digits that are all different and that increase from left to right?

ANSWERS TO SUPPLEMENTAL PROBLEMS

LEVEL 1: NUMBER THEORY

1. E
2. B
3. D
4. A
5. C
6. 81
7. 9.2
8. 202

LEVEL 1: ALGEBRA AND FUNCTIONS

9. C
10. D
11. A
12. E
13. A
14. 4/3 or 1.33
15. 5/3, 1.66 or 1.67
16. 11

LEVEL 1: GEOMETRY

17. D
18. A
19. D
20. 28
21. 74
22. 92
23. 16
24. 50

LEVEL 1: PROBABILITY AND STATISTICS

25. E
26. A
27. C
28. 2/17, .117, or .118
29. 28
30. 537
31. 16
32. 25

LEVEL 2: NUMBER THEORY

33. A
34. B
35. E
36. C
37. D
38. 35
39. 24
40. 18

LEVEL 2: ALGEBRA AND FUNCTIONS

41. C
42. D
43. A
44. B
45. B
46. 10
47. 4
48. 44

LEVEL 2: GEOMETRY

49. A
50. C
51. D
52. B
53. C
54. 34
55. 6
56. 360

LEVEL 2: PROBABILITY AND STATISTICS

57. E
58. B
59. C
60. C
61. 72
62. 22
63. 11
64. 16

LEVEL 3: NUMBER THEORY

65. E
66. C
67. D
68. D
69. 6
70. 105
71. 20
72. 1024

LEVEL 3: ALGEBRA AND FUNCTIONS

73. B
74. E
75. E
76. C
77. E
78. E
79. 625
80. 1

LEVEL 3: GEOMETRY

81. D
82. D
83. B
84. B
85. 3.2
86. 62.5
87. 6
88. 100

LEVEL 3: PROBABILITY AND STATISTICS

89. D
90. A
91. D
92. 11
93. 3
94. 1/2 or .5
95. 720
96. 11

LEVEL 4: NUMBER THEORY

97. E
98. C
99. C
100. B
101. A
102. E
103. 1
104. 96

LEVEL 4: ALGEBRA AND FUNCTIONS

105. D
106. E
107. B
108. 12
109. 8
110. 3
111. 21
112. 64

LEVEL 4: GEOMETRY

113. E
114. E
115. A
116. 45/2 or 22.5
117. 18
118. 57.8
119. 4
120. 32

LEVEL 4: PROBABILITY AND STATISTICS

121. C
122. E
123. D
124. C
125. 120
126. 56
127. 27
128. 45

LEVEL 5: NUMBER THEORY

129. B
130. A
131. E
132. D
133. E
134. 38
135. 33
136. 5

LEVEL 5: ALGEBRA AND FUNCTIONS

137. D
138. C
139. E
140. B
141. C
142. 23
143. 12.3
144. 12

LEVEL 5: GEOMETRY

145. C
146. C
147. E
148. A
149. A
150. 20
151. 17/4 or 4.25
152. 11.1, 11.2, 11.3, 11.4 , 11.5, 11.6, 11.7, 11.8, or 11.9

LEVEL 5: PROBABILITY AND STATISTICS

153. E
154. C
155. D
156. A
157. B
158. 6
159. 3/2 or 1.5
160. 5